

Answer keys

1	C	2	B	3	A	4	A	5	A	6	B	7	A
8	B	9	C	10	D	11		12	C	13	D	14	A
15		16	D	17	C	18		19	B	20	A	21	A
22	A	23		24	A	25	C	26	A	27	D	28	B
29	A	30	A	31	D	32	A	33	D	34	B	35	A
36	C	37	A	38	D	39	C	40	C	41	C	42	A
43	B	44	B	45	C	46	B	47	B	48	C	49	C
50	B	51	C	52		53	B	54	D	55	A	56	C
57	D	58		59	B	60	C	61	A	62	D	63	C
64		65		66		67	A	68	C	69	B	70	
71	D	72	C	73	B	74	D	75		76	C	77	A
78	B	79	C	80	A	81	D	82	D	83	C	84	C
85	D												

Explanation:

1. Use $[\lambda I - P] = 0$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} = 0$$

$$\begin{bmatrix} \lambda - P_{11} & -P_{12} \\ -P_{21} & \lambda - P_{22} \end{bmatrix} = 0$$

$$\Rightarrow (\lambda - P_{11})(\lambda - P_{22}) - P_{21}P_{12} = 0$$

Putting $\lambda=0$, we get $P_{11}P_{22} - P_{21}P_{12} = 0$

2. $4x + 2y = 7 \rightarrow (1)$

$2x + y = 6 \rightarrow (2)$

(1) can be written as $2x + y = 3.5 \rightarrow (3)$

Now LHS of equation (2) and (3) are same but RHS is different which is not possible. Hence no solution

3. Sine value lies between -1 and +1. Therefore no real or complex solution exists

4. $f(x) = e^x + e^{-x}$
 $f'(x) = e^x - e^{-x}$
 $f'(x) = 0 \Rightarrow e^x - e^{-x} = 0$
 $f''(x) = e^x + e^{-x} \Rightarrow +ve \text{ for } x = 0$
 Thus minimum.
 Minimum $f(x) = e^0 + e^0 = 2$

5. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$
 $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$
 Thus $\sin(x^3)$ will have odd powers
 $\sin(x^2)$ will have even powers
 $\cos(x^3)$ will have even powers
 $\cos(x^2)$ will have even powers

6. $\frac{dx(t)}{dt} + 3x(t) = 0$
 $\int \frac{dx(t)}{x(t)} = \int 3dt$
 $\log x(t) = 3t$
 $x(t) = k.e^{-3t}$
 Thus option (B) satisfies the solution

8. At $t = 0$, inductor acts like a open circuit
 We know $L \frac{di}{dt} = V$
 at $t=0^+$, inductor path is open circuit
 $V = I_S.R_S$ (at $t = 0^+$)
 Thus $\frac{di}{dt} = \left(\frac{I_S.R_S}{L} \right)$

9. Only option (C) satisfies the condition of causality

10. $h(t) = e^{+\alpha t}u(t) + e^{\beta t}u(-t)$
 For $h(t)$ to be stable $\int h(t) dt < \infty$
 It will happen when α is negative and β is positive.

16. Curl of electric field is zero.
Divergence of magnetic field is zero.

17. During +ve part of V_i
 D_1 will be forward biased
Zener diode is reverse biased
Thus net voltage = $6.8 + 0.7 = 7.5V$
During -ve part of V_i
 D_2 will be forward biased
 D_1 will be reverse biased
Thus net voltage = $-0.7V$

19. $I_D = k(V_{GS} - V_T)^2$, $\frac{\delta I_D}{\delta V_{GS}} = 2k(V_{GS} - V_T) = gm$

20. $A_C \cos \omega_C t + 2 \cos \omega_m t \cos \omega_C t$

$$A_C \cos \omega_C t \left[1 + \frac{2}{A_C} \cos \omega_m t \right]$$

for envelop detection $\mu < 1 \Rightarrow \frac{2}{A_C} < 1 \Rightarrow A_C$ should be at least 2

21. For finding Thevenin equivalent
Short voltage source
Open circuit source

$$\text{Now Thevenin equivalent} = \left(1 + \frac{1}{S}\right) \parallel (1 + S) = \frac{1 + \frac{1}{S} + S + 1}{1 + \frac{1}{S} + 1 + S} = 1$$

22. $Y = R + \frac{1}{CS} + SL$

$$Z = \frac{CS}{S^2 LC + RCS + 1}$$

Comparing it with given equation

$$\frac{0.25}{S^2 + 0.1S + 2} = \frac{0.1S}{\frac{1}{2}S^2 + 0.05S + 1}$$

$$C = 0.1$$

$$RC = 0.05$$

$$R = 0.5$$

$$LC = \frac{1}{2}$$

$$L = \frac{1}{0.2} = 5$$

24. Cumulative distribution function = $\int_{-\infty}^x (\text{Pdf}) dx$

Thus when we integrate the line from (-1, 0) to (0, 1) we get a parabolic curve.

The maximum value of Pdf can be 1

Thus option (A) satisfies the solution

25. $f(x_n) = x_n e^{-x_n}$

$f'(x_n) = 1 + e^{-x_n}$

$$x_{n+1} = x_n - \frac{x_n - e^{-x_n}}{1 + e^{-x_n}} = \frac{x_n + x_n e^{-x_n} - x_n + e^{-x_n}}{1 + e^{-x_n}} = \frac{(x_n + 1) e^{-x_n}}{1 + e^{-x_n}}$$

26. Residue at $z = 2$

$$\lim_{z \rightarrow 2} \frac{1}{1!} \frac{d}{dz} (z-2)^2 \frac{1}{(z+2)(z-2)^2} \Rightarrow \text{Residue} = \frac{-1}{32}$$

27. $P = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

find $L^{-1}[SI - A]^{-1}$

$$\left[\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \right]^{-1}$$

$$\begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}^{-1}$$

$$L^{-1} \left[\frac{1}{(S+1)(S+2)} \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix} \right] \text{ find inverse laplace transform of the expression}$$

28. $f(x) = e^x + \sin x$

Coefficient of $(x-\pi)^2 = \frac{1}{2!} f''(x)$

$f'(x) = e^x + \cos x$

$f''(x) = e^x - \sin x$

$f''(x)|_{x=\pi} = e^\pi$

Thus coefficient of $(x-\pi)^2 = 0.5$

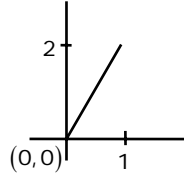
29. Using property of probability density of $\int_{-\infty}^{\infty} P_x dx = 1$

$$\int_{-\infty}^{\infty} [M \exp(-2|x|) + N \exp(-3|x|)] dx = 1 \Rightarrow M + \frac{2}{3}N = 1$$

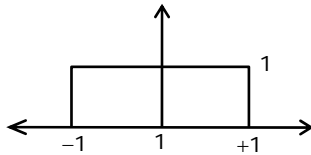
30. $\int_C (4x^3 + 10y^4)$

$y = 2x$

$\Rightarrow \int_0^1 [4x^3 + 10(2x)^4] dx = 33$



32.



$$X(j\omega) = \int_{-1}^1 1 \cdot e^{-j\omega t} dt = \frac{2 \sin \omega}{\omega}$$

At $\omega=0$ $X(j0) = 2$

At $\omega = \pi$ $X(j\pi) = 0$

$\omega = 2\pi$ $X(j2\pi) = 0$

33. Use convolution to get the result

35. R_1 $y(t) = t^2x(t)$

Linear time variant

R_2 $y(t) = t(x(t))$

Non linear non time invariant

R_3 $y(t) = (x(t))$

Non linear time invariant

R_4 $y(t) = x(t - 5)$

Linear time invariant

36. $H = nP \log \frac{1}{P}$

Since they all have same probability

Thus it increases with n

37. Direct property

Convolution in time domain \leftrightarrow

Multiplication in frequency domain

$$38. \quad P = \frac{25}{S^2 + 25}, \quad \zeta = 0, \quad R = \frac{36}{S^2 + 12S + 36}, \quad \zeta = 1$$

$$Q = \frac{36}{S^2 + 20S + 36}, \quad \zeta = \frac{20}{12} > 1, \quad S = \frac{49}{S^2 + 7S + 49}, \quad \zeta < 1$$

39. Closed loop system transfer function

$$C(S) = \frac{S + 8}{S^2 + S(\alpha + 1) + 4}$$

Use Routh criteria

$$\begin{array}{l|ll} S^2 & 1 & 4 \\ S & (\alpha + 1) & 0 \\ S^0 & 4 & \end{array}$$

Thus for all positive value of ' α ' this will be stable

$$40. \quad \frac{dx_1}{dt} = -\alpha x_1 + \beta x_2 + u$$

$$\frac{dx_2}{dt} = -\beta x_1 + \gamma x_2 + u$$

$$\frac{dx_3}{dt} = \alpha x_1 + \gamma x_2 + 0.u$$

Thus option (C) satisfies the solution

41. Use Routh criteria.

42. Satisfy the given condition with different option given

Option (A) satisfy the solution

$$43. \quad \frac{V_o}{V_i} = -\left(\frac{Z}{Z_1}\right)$$

$$Z_1 = R_1 \parallel \frac{1}{SC_1} = \left(\frac{R_1}{R_1 C_1 S + 1}\right)$$

Now first case

$$Z = R_2 + \frac{1}{SC_2} = \left(\frac{R_2 C_2 S + 1}{SC_2}\right)$$

$$\frac{V_o}{V_i} = \frac{(R_2 C_2 S + 1)(R_1 C_1 S + 1)}{R_1 C_2 S}$$

Thus it is PID controller

Second case

$$Z = R_2 \parallel \frac{1}{SC_2} = \left(\frac{R_2}{R_2 C_2 S + 1} \right)$$

$$\text{Thus } \frac{V_o}{V_i} = \left(\frac{R_1 C_1 S + 1}{R_2 C_2 S + 1} \right) \left(\frac{R_2}{R_1} \right)$$

Taking phase = $\tan^{-1} \omega R_1 C_1 - \tan^{-1} \omega R_2 C_2$

Given that $R_1 C_1 < R_2 C_2$

Thus phase is -ve

Hence this is phase lag compensator

44. Given circuit is current mirror

$$I_G = 0$$

$$I_x = I_{\text{bias}}$$

47. Given circuit is low pass filter

50. Charging and discharging level of capacitor will be the voltage across it

This is equal to $\frac{1}{3} V_{CC}$ and $\frac{2}{3} V_{CC}$

Thus 3V to 6V is the voltage V_C across the capacitor

51. Use $E_F - E_V = \frac{kT}{q} \ln \left(\frac{N_V}{N_A} \right)$

Since it is doped with acceptor impurity, Fermi level will shift down

53. V_o is switching between +15 to -150

Thus voltage at non inverting input switches between $\frac{15 \times 10}{10 + 10}$ to $\frac{-15 \times 10}{10 + 10}$, 7.5 to -7.5

54. AND gate

56. $P = 11101101 = -25$

$Q = 11100110 = -18$

Thus $P - Q = -25 - (-18) = -7$

Thus '-7's two complement representation is 11111001

57. $M_1 = Z \text{ XOR } R$

$$M_1 = (X.Y) \text{ XOR } R$$

$$M_1 = [\overline{P.Q}.(P+Q)] \text{ XOR } R = (P\overline{Q} + PQ) \text{ XOR } R = (P \text{ XOR } Q) \text{ XOR } R$$

58. Use this to find solution

R S Z

0 0 P + \overline{Q}

0 1 P

1 0 P.Q

1 1 P

59. Two JK flip flop. Thus propagation delay will be $2\Delta T$
Frequency is getting reduced by a factor e

$$61. f = \frac{3 \times 10^8}{2} \times \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{3}\right)^2} = \frac{3 \times 10^8 \times 5}{2 \times 4 \times 3 \times 10^{-2}} = 6.25 \text{ GHz}$$

$$62. Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta \ell}{Z_0 + jZ_L \tan \beta \ell} \right)$$

$$X = 10 \text{ cm}, Z_L = 0, \beta \ell = \frac{2\pi}{10} \times 1$$

$$\text{Thus } Z_{in} \text{ is } Z_0 \left(\frac{0 + jZ_0 \tan \frac{\pi}{5}}{Z_0} \right) \text{ which is inductive}$$

$$63. n_2 = \frac{1}{3} \sqrt{\frac{\mu_0}{\epsilon_0}}, n_1 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\text{Thus reflection coefficient} = \frac{\frac{1}{3} - 1}{\frac{1}{3} + 1} = -0.5$$

Thus magnitude is 0.5

67. Probability of error = P

Thus probability of no error = (1-P)

Now probability that transmitted bit, received in error

= all bits are with error + one bit is with error

$$= P^3 + 3C_1 P^2 (1 - P) = P^3 + 3P^2 (1 - P)$$

68. In TDM minimum bandwidth is twice the maximum frequency present

Thus 6W is the answer

69. Total frequency deviation = $5 \times 1500 + 7.5 \times 1000 = 15000$

Thus modulation index = $\frac{\Delta f}{f_m / \max} = \frac{15000}{1500} = 10$

71. According to Nyquist Criteria, minimum sampling frequency = $2 \times 4k = 8\text{kHz}$

But each cycle can accommodate two bits

Thus minimum frequency = 4 kHz

72. SNR = $6n + 1.7 = 6 \times 8 + 1.7 = 48 + 1.7$. Thus correct answer is (C)

73. To reduce quantization noise by 4

No. of levels should increase by 2

Thus 512 levels are required.

74.
$$V_c(t) = \frac{1/SC}{1/SC + SL + R}$$

$$= \frac{1/S}{1/S + S + 1} = \frac{1}{S^2 + S + 1} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}/2}{(S + 1/2)^2 + (\sqrt{3}/2)^2} = \frac{2}{\sqrt{3}} e^{-1/2t} \sin \frac{\sqrt{3}}{2} t$$

75. Similar way this question can be solved

76. $V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

Using the given information: S_1 -open, S_2 -closed

$4.5 = Z_{12}1 \Rightarrow Z_{12} = 4.5$

$1.5 = Z_{22}1 \Rightarrow Z_{22} = 1.5$

S_1 -closed, S_2 -open

$6 = Z_{11}1 \Rightarrow Z_{11} = 1.5$

$6 = Z_{21}4 \Rightarrow Z_{21} = 1.5$

Thus Z matrix = $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 1.5 \end{bmatrix}$

77. $V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow (1)$

$$I_2 = \frac{-Z_{21}I_1 + V_2}{Z_{22}} \rightarrow (2)$$

$$I_2 = \frac{-Z_{21}}{Z_{22}}I_1 + \frac{1}{Z_{22}}V_2 \rightarrow (2)$$

Substitute (2) in (1) to get V_1 in terms of I_1 and V_2

$$\text{Thus H matrix} = \begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$$

78. Initial voltage across R = 5Vn

Final voltage across R = 0V

$$\text{Thus } V_R = 5.e^{-0.5t}$$

When we sample this at $f_s = 102$

$$\text{We get } x(n) = 5.e^{-\frac{0.5}{10}n} = 5.e^{-0.05n}$$

79. $X(Z) = \sum_{n=0}^{\infty} 5.e^{-0.05n}.Z^{-n}$

$$= \frac{5}{1 - e^{-0.05}Z^{-1}} \quad |Z| > e^{-0.05}$$

$$= \frac{5Z}{Z - e^{-0.05}} \quad |Z| > e^{-0.05}$$

80. Design is independent of I_B as β is very high

$$I_\beta \approx 0$$

$$V_\beta = \frac{9 \times 10}{20 + 10} = 3V$$

Thus applying Kirchhoff's Law in base emitter, we get

$$3 = 0.7 + I_E \times 2.3 \times 10^3 \Rightarrow I_E = 1 \text{ mA}$$

81. Mid band voltage gain = $-\frac{R_L'}{r_e}$

$$r_e = \frac{25}{I_E} = 25 \Omega$$

$$R_L' = 3k \parallel 3k = R_C \parallel R_L = 1.5k$$

$$\text{Thus } A_V = -\frac{1500}{25} = -60$$

82.	$b_3b_2b_1b_0$	V_{DAC}
	0 0 0 0	0
	0 0 0 1	0.5
	0 0 1 1	15
	⋮	⋮
	1 1 0 1	6.5
	1 1 1 0	7
	1 1 1 1	7.5

Using the given V_{DAC} relation we can get t_c

Thus stable reading of LED is $6.5 \times 2 = 13$

83. Magnitude error = $6.5 - 6.2 = 0.3$

84.
$$H(\omega) = \int_0^{\infty} e^{-2t} e^{-j\omega t} dt = \left(\frac{1}{j\omega + 2} \right)$$

85. When input is sinusoidal then output is also sinusoidal with same frequency but amplitude and phase changes.

Thus amplitude is $2 \left| \frac{1}{j\omega + 2} \right|_{\text{at } \omega=2} = \frac{2}{2\sqrt{2}} = 2^{-0.5}$

Phase is $\tan^{-1} \left(\frac{\omega}{2} \right) \Big|_{\omega=2} = 0.25\pi$

Thus $y(t) = 2^{-0.5} \cos(2t - 0.25\pi)$